Decoupling of a Class of Nonlinear Systems and Its Application to an Aircraft Control Problem

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The necessary and sufficient condition for decoupling a nonlinear system with state feedback is obtained. It is shown that when this condition is satisfied there exists a control law which makes each output variable of the dynamical system independently controllable with a separate input. The theory is then applied to an aircraft control problem where the implication of the theoretical results is discussed. The objective of the aircraft control problem is to decouple the vertical and the horizontal path angles of the flight trajectory relative to earth-fixed axes. The aircraft equations are simplified by postulating a rudder control law which maintains zero sideslip velocity in flight. The control laws for the elevator and aileron which decouple the simplified aircraft model are obtained. The control system is then evaluated in a simulation study to show that it indeed decouples the flight path angles.

Nomenclature

x, z = n-dimensional state and transformed state
u, y = m-dimensional control and output
w, v = m-dimensional internal and external command inputs
$f(x), B(x), C$ = Matrices associated with the dynamical system
$D(x)$ = Matrix which determines if the system can be decoupled
$h(x), G(x)$ = Decoupling pair of matrices
$Q(x)$ = Transformation of the state
K = Feedback and Feedforward gain matrices
I = Identity matrix
a_i, b_i, k_{ij}, etc. = Constant gains
α, β = Angle of attack and sideslip
ϕ = Roll angle
p, q, r = Angular rates about the x, y, z body axes
q_w, r_w = Wind referenced pitch and yaw rate
$\delta_e, \delta_a, \delta_r$ = Aircraft control inputs, elevator, aileron and rudder
g, V_o = Gravitational constant and airspeed
γ, ψ = Vertical and horizontal flight path angles
ξ, ζ, μ, ν = Auxiliary functions defined in Eq. (26)
M_q, L_r, etc. = Normalized stability derivatives Z, Y, L, M, N are stability derivatives associated with lift, sideslip, rolling moment, pitching moment and yawing moment equations, respectively.
$\bar{M}_q, \bar{L}_r$, etc. = Constants related to the normalized stability derivatives

Subscripts

c = Command input
j = j'th row or i'th element

Notations

$\nabla_x f(x)$ = Matrix of partial derivatives of $f(x)$ with respect to x
$\dot{y}_i, \overset{(j)}{y_i}$ = first and j'th time derivative of y_i
$x \in X$ = For all x which are included in the domain of definition X
R^n = n-dimensional state space

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1. Introduction

IN order to maintain constant altitude in turning flight the aircraft pilot must apply control inputs to the aileron and to the elevator simultaneously. The aileron causes the aircraft to roll and make a banked turn, while the elevator increases the angle of attack which compensates for the loss of vertical lift that occurs during the banked turn. In many applications it is desirable to design a control system that will automatically blend the elevator and aileron deflections to alleviate the pilot from tedious task of controlling two variables simultaneously. With such an automatic control system the pilot will have at his disposal controls for the vertical and horizontal flight path angles, such that activating either control will affect only one flight path angle. A control system that provides such independent control of the output variables with separate command inputs is said to decouple the dynamical system. It is shown in the appendix that the flight path angles are nonlinear functions of the aircraft motion variables, therefore, in order to control the flight path angles independently one must decouple a nonlinear system.

The problem of decoupling a linear dynamical system has been investigated in great detail.[1-8] Morgan[1] first posed the linear decoupling problem. Falb and Wolovich[2] established the necessary and sufficient conditions for decoupling and Gilbert[3,4] presented the complete structure of the decoupling control law. Gilbert's method, however, has a major drawback. It does not always yield a stable decoupled system. Recently, Wonham and Morse[5-7] looked at the decoupling problem within a geometric framework. Through the geometric theory, which is free from matrix algebra, they have extended Gilbert's[3] results. In addition, they have developed the "Extended Decoupling Problem" which relieves the difficulty of possibly obtaining an unstable decoupled system through Gilbert's[3] method. Despite the abstract nature of the results in Refs. 5-7, the geometric theory has been successfully applied to a linear aircraft control problem in Cliff and Lutze.[8]

At the time the study, which is summarized in this paper, was performed there was no other work reported in the literature on the decoupling of multivariable nonlinear systems. Since then some related work has appeared in current journals. Nazar and Rekasius[9] have obtained a sufficient condition for the decoupling of a very specific class of nonlinear systems. The existence of a decoupling

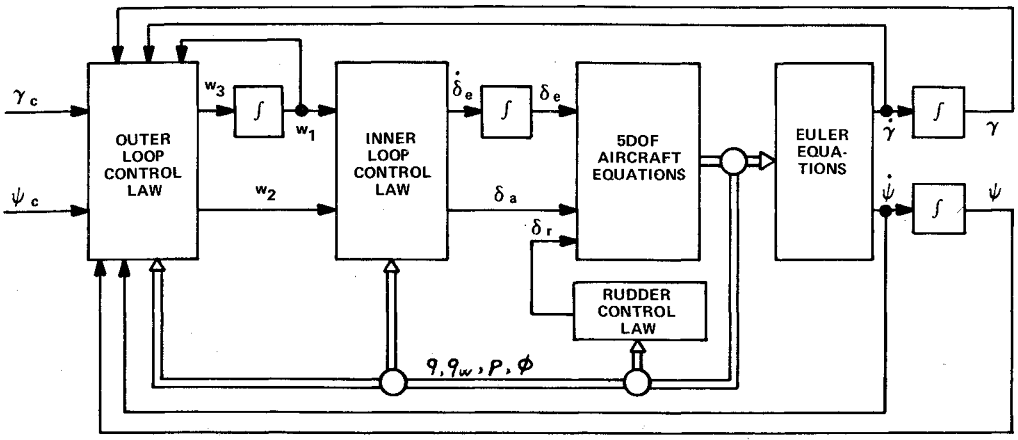

Fig. 1 Block diagram showing the inner-loop and outer-loop control systems.

control law, which is based on the "Reduced Linear System" portion of the nonlinear system, is quite similar to the existence of a decoupling control law for linear systems.[2,3] This differs considerably from the results obtained in this paper and in Singh and Rugh[10] where the existence of a decoupling control law is based on the entire nonlinear system. Singh and Rugh[10] formulated a nonlinear decoupling problem which is quite similar to that developed here. They have obtained necessary and sufficient conditions for the decoupling of a time-varying nonlinear system in which the output is also a nonlinear function of the states. Despite the generality of the method in Ref. 10, it does not lend itself easily to the solution of practical control problems. The results obtained here are easier to follow and readily applicable to practical control problems.

In this paper the necessary and sufficient conditions for the decoupling of a class of nonlinear systems will be obtained and a sufficiently general decoupling control law will be constructed. The theory will then be applied to an aircraft control problem where the implication of decoupling will be discussed. The theory will be first applied to a simplified (ideal) aircraft model. The results of a simulation study will then be presented to show that the control law derived for the ideal aircraft produces nearly decoupled response when used in conjunction with the real aircraft equations. The aircraft control problem discussed here is of considerable practical importance.[11] The design of a control system that decouples the flight path angles of the aircraft flight trajectory would not have been possible without the proper tools for decoupling a nonlinear system.

Formulation of the Nonlinear Decoupling Problem

The objective here is to find the control law that decouples the class of nonlinear systems which evolve according to

$$\dot{x} = f(x) + B(x)u$$
$$y = Cx \tag{1}$$

where x is the n-dimensional state, u is the m-dimensional control, $(m \leq n)$ and y is the m-dimensional output. The elements of $f(x)$ and $B(x)$ are real nonlinear functions of the state x. $f(x)$ is continuously differentiable with respect to x and C is a constant matrix. A constant matrix C implies that each output y_i is a linear combination of the states. A more general formulation in which the output is a nonlinear function of the states is given in Singh and Rugh.[10]

The class of control laws of interest are those of the form

$$u = h(x) + G(x)w \tag{2}$$

where $h(x)$ is $mx1$ matrix representing the state feedback control law, $G(x)$ is mxm matrix of feedforward gains and w is the m-dimensional command input vector. The system is defined in a domain $X \in R^n$ where X is an open set, such that a unique solution of Eq. (1) exists for any initial condition $x(t_0) \in X$ and for a piecewise continuous control u. Moreover, $G(x)$ is assumed to be nonsingular for all $x \in X$.

Definitions

The purpose of decoupling is to find the matrices $h(x)$ and $G(x)$ such that when Eq. (2) is substituted into Eq. (1) it produces a closed-loop system in which each input w_i influences one and only one output y_i.

The following definitions are basic to the formulation of this problem. The nxn matrices $A_1(x), \ldots$ are generated from $f(x)$ by the successive gradient operation.

$$\begin{aligned} A_0(x) &= I \\ A_1(x) &= \nabla_x(f(x)) \\ A_2(x) &= \nabla_x(A_1(x)f(x)) \\ A_{k+1}(x) &= \nabla_x(A_k(x)f(x)) \end{aligned} \tag{3}$$

where $\nabla_x f(x)$ denotes the gradient of $f(x)$ with respect to x.

$$\nabla_x f(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots\cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots\cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Now compute the integer constants $d_1 \ldots d_m$ as follows: Form the row vector $C_i B(x)$, where C_i is the i'th row of C. If this row vector is zero for all $x \in R^n$ then form the row vector $C_i A_1(x)B(x), \ldots$ etc., until the first nonzero row vector $C_i A_j(x)B(x) \neq 0$ is obtained. The integer d_i is the smallest integer j such that the row vector $C_i A_j(x)B(x)$ is nonzero for all $x \in X$. Notice that the parameters d_i's thus defined do not depend on x. Now construct the mxm matrix $D(x)$ and the $mx1$ vector $f^*(x)$

$$D(x) = \begin{bmatrix} D_1(x) \\ \vdots \\ D_m(x) \end{bmatrix}, \qquad f^*(x) = \begin{bmatrix} f_1^*(x) \\ \vdots \\ f_m^*(x) \end{bmatrix}$$
$$D_i(x) = C_i A_{d_i}(x)B(x), \qquad f_i^*(x) = C_i A_{d_i}(x)f(x) \tag{4}$$

where D_i and f_i^* denote the i'th rows of D and f^*, respectively. Using these definitions we will first obtain the necessary and sufficient condition for the control law (2) to decouple the nonlinear system, Eq. (1), we will then find a more general characterization of the control law.

Theorem 1

There exists a control law, Eq. (2), where $G(x)$ is nonsingular for all $x \in X$, that decouples the system, Eq. (1) on X if and only if the matrix $D(x)$ is nonsingular for all $x \in X$.

Sufficiency

Using the definition of the parameters d_i, $i=1,\ldots m$ we obtain the successive derivatives of the output variable y_i.

$$y_i = C_i x$$
$$\dot{y}_i = C_i f(x)$$
$$\vdots$$
$$y_i^{(d_i)} = C_i A_{d_i-1}(x) f(x) \quad (5)$$
$$y_i^{(d_i+1)} = C_i A_{d_i}(x) f(x) + D_i(x) u \quad (6)$$

If $D(x)$ is nonsingular for all $x \in X$ it is claimed that the pair of matrices

$$h(x) = -D^{-1}(x) f^*(x)$$
$$G(x) = D^{-1}(x) \quad (7)$$

decouples Eq. (1) for all $x \in X$. In fact, substituting Eqs. (2), (4), and (7) into Eq. (6) yields

$$y_i^{(1+d_i)} = w_i \quad (8)$$

which shows that y_i depends only on w_i and therefore, the pair of matrices $h(x)$ and $G(x)$ in Eq. (7) decouples the system. This proves sufficiency.

Necessity

Now suppose that there exists a pair of matrices $h(x)$ and $G(x)$ which decouple Eq. (1), such that $G(x)$ is nonsingular for all $x \in X$. From Eq. (6) one has

$$y_i^{(1+d_i)} = C_i A_{d_i}(x) f(x) + D_i(x) h(x) + D_i(x) G(x) w \quad (9)$$

Then one must have

$$D_i(x) G(x) = \sigma_i(x) E_i \qquad i = 1,\ldots m \quad (10)$$

where $\sigma_i(x)$ is a real valued function of x and E_i is the i'th row of the mxm identity matrix. For, if Eq. (10) did not hold, y_i would depend on w_j for $i \neq j$ and the system would not be decoupled. Moreover, $\sigma_i(x) \neq 0$ for all $x \in X$, since $D_i(x) \neq 0$ for all $x \in X$ by the definition of d_i, $i = 1, \ldots m$. If Eq. (10) holds for every $i = 1, \ldots m$ then one has

$$D(x) G(x) = \mathrm{Diag}[\sigma_1(x), \ldots \sigma_m(x)]$$

and therefore, $D^{-1}(x)$ must exist in the domain X since $G(x)$ is nonsingular in this domain. This completes the proof of the theorem.

For a linear system the matrices in Eq. (3) reduce to $A_j = A^j$; $j = 0, \ldots d_i$. Hence the proof given in Falb and Wolovich[2] is a special case of the theorem just proved. A more general proof for the class of time-varying nonlinear systems described by $\dot{x}(t) = A(x,t) + B(x,t)u(t)$, $y(t) = C(x,t)$ is given in Singh and Rugh.[10] The operator $L_A{}^k(C_i)$ defined in Ref. 10 reduces to $L_A{}^k(C_i) = C_i A_{k-1}(x) f(x)$ for a time-invariant nonlinear system with a constant C, where the matrices $A_k(x)$ are those defined in Eq. (3).

Structure of the Control Law

It has been shown that the pair of matrices $h(x)$ and $G(x)$ in Eq. (7) decouples the system, Eq. (1) for all $x \in X$ and yields the decoupled system, Eq. (8) in which each output y_i evolves in time according to the $1 + d_i$th fold integral of the input w_i. This structure of the closed-loop system is unsatisfactory. Therefore, a more general characterization must be found for the control law in order to obtain a desirable closed-loop structure in addition to achieving decoupling. A sufficiently general form for the control law w is obtained by the following construction.

Let $Q(x)$ be an n-vector valued transformation on x defined by

$$Q^T(x) = [Q_1{}^T(x), Q_2{}^T(x), \ldots Q_m{}^T(x), Q_{m+1}{}^T(x)]$$

where

$$Q_i{}^T(x) = [C_i x, C_i f(x), C_i A_1(x) f(x), \ldots C_i A_{d_i-1}(x) f(x), \bar{Q}_i{}^T(x)]$$
$$i = 1, \ldots m \quad (11)$$

The dimension of each $z_i = Q_i(x)$ is $p_i \times 1$ such that $\sum_{i=1}^{1+M i} p_i = n$, and the dimension of each $\bar{Q}_i(x)$ is $r_i \times 1$ where $r_i = p_i - (1 + d_i)$, $i = 1, \ldots m$. Now assume that the new state vector obtained by the transformation $z = Q(x)$ has the following properties. Each $z_i = Q_i(x)$ is the maximal set of state variables which can be influenced by one and only one command input w_i, while $z_{m+1} = Q_{m+1}(x)$ contains the elements of z which are either coupled to two or more subvectors z_i and z_j, $i \neq j$ or decoupled from all z_i, $i = 1,\ldots m$. It is clear from the construction of $Q(x)$ that each z_i is decoupled from z_j for $i \neq j$; $i,j = 1, \ldots m$. The class of control laws for w of interest here are of the form

$$w = Kz + \Lambda v \quad (12)$$

where K and Λ are constant matrices of appropriate dimensions and v is an m-dimensional external command input. We will now state and prove a theorem which will establish the basis for the general characterization of the control law.

Theorem 2

The decoupling property of the system in Eq. (8) is invariant with the control law, Eq. (12) if (sufficiency condition) K and Λ have the following structure

$$K = \begin{bmatrix} k_1 & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & k_m & 0 \end{bmatrix}_{\underbrace{}_{p_{m+1}}} ; \quad k_i \rightarrow 1 \times p_i \quad (13)$$

$$\Lambda = \mathrm{Diag}(\lambda_1, \ldots\ldots \lambda_m)$$

Proof

From Eqs. (5) and (11) one calculates the d_ith element of z_i as $z_{id(i)} = C_i A_{d(i)} - 1\, x f(x) = y_i d_i$. Now substituting Eq. (13) into Eq. (12) and Eq. (12) into Eq. (8) and recognizing that $\dot{z}_{id(i)} = y_i^{(1+d_i)}$ one gets $\dot{z}_{id(i)} = k_i z_i + \lambda_i v_i$ where each v_i influences only one subvector z_i. Therefore, the control law, Eqs. (12) and (13) preserves decoupling in Eq. (8). Note that a diagonal matrix Λ is also necessary, while K may contain additional nonzero entries and still preserve decoupling in Eq. (8).

The problem of constructing the vector z for a nonlinear system with the aforementioned properties is unsolved.

Specifically, there is no known method for constructing the r_i – vector valued functions $\bar{Q}_i(x)$. Therefore, we confine the results of the present analysis to a less general control structure for w, by simply deleting the $\bar{Q}_i(x)$, $i = 1, \ldots m$, and reducing the dimension of each z_i to $1 + d_i$. With this provision, substituting Eqs. (7), (11), and (12) into Eq. (2) yields the control law for u.

$$u = D^{-1}(x)[KQ(x) - f^*(x) + \Lambda v] \tag{14}$$

where

$$Q(x) = \begin{bmatrix} Q_1(x) \\ \vdots \\ Q_m(x) \end{bmatrix};\quad Q_i(x) = \begin{bmatrix} C_i x \\ C_i f(x) \\ \vdots \\ C_i A_{d_{i-1}}(x) f(x) \end{bmatrix};$$

$$\Lambda = \mathrm{Diag}(\lambda_1 \ldots \lambda_m) \qquad K = \begin{bmatrix} k_1 & & 0 \\ & \ddots & \\ 0 & & k_m \end{bmatrix}$$
$$k_i = [-k_{io}, \ldots -k_{id_i}]$$

Here $Q(x)$ is $p = \Sigma_{i=1}^{m}(1 + d_i)$ dimensional vector, K is mxp dimensional constant matrix and each k_i is $(1 + d_i)$ dimensional constant row vector.

Notice that the control law just derived yields a closed-loop system in which each output y_i evolves according to a linear differential equation

$$y_i^{(1+d_i)} + k_{id_i} y_i^{(d_i)} + \ldots k_{io} y_i = \lambda_i v_i \tag{15}$$

where the gains k_{ij} are picked at will to obtain a desired output response. The state variables of the closed-loop system, on the other hand, evolve according to nonlinear differential equations [just substitute Eq. (14) into Eq. (1)]. Very little can be said about the stability of the overall nonlinear system, therefore, it is usually necessary to conduct a stability analysis to ascertain the usefulness of the control law obtained on the basis of decoupling.

Decoupling the Vertical and Horizontal Motions of an Aircraft

The theory will now be applied to an aircraft control problem in which it is desired to construct a control law that decouples the vertical (γ) and the horizontal (ψ) path angles of the flight trajectory. The equations which describe the motion of the aircraft relative to earth axes are given in the appendix by Eqs. (A1–A3). Equation (A1) gives the aircraft equations of motion in body coordinates. The motion variables measured along the body axes are first transformed to wind axes in Eq. (A3) and then transformed to earth axes through the Euler transformation equations given in Eq. (A2).

The design of a control system is not a simple matter of applying the decoupling theory to the nonlinear set of Eqs. (A1–A3). In fact, if one applies the theory directly to these equations one gets a matrix $D(x)$.
$D(x)$.

$$D(x) = \begin{bmatrix} -Z_{\delta_e} \sin\phi & 0 \\ -Z_{\delta_e} \cos\phi & 0 \end{bmatrix}$$

which is singular for all $x \in R^n$. Therefore, the real aircraft model represented by Eqs. (A1–A3) cannot be decoupled. In order to circumvent this difficulty, a decoupling control law will be obtained for a simplified (ideal) aircraft model. This control law will then be used with the real aircraft equations in a simulation study to determine how well it decouples the vertical and the horizontal motions of the real aircraft. The simplified aircraft model is obtained in the appendix by assuming that an automatic rudder control system maintains zero sideslip angle during the entire flight regime. The resulting ideal aircraft equations are given by Eqs. (A6–A9). These equations are rewritten here by omitting the bars for simplicity.

Aircraft Equations in Body/Wind Axes

$$\begin{aligned} \dot{q}_w &= Z_\alpha(q - q_w) Z_{\delta_e} \cdot \dot{\delta}_e + \frac{g}{V_0} p \sin\phi \\ \dot{q} &= M_q q + M_{qw} q_w + M_{\delta_e}\delta_e + M_\phi(\cos\phi - 1) \\ \dot{p} &= L_p p + L_\phi \sin\phi + L_{\delta_a} \cdot \delta_a \\ \dot{\phi} &= p \end{aligned} \tag{16}$$

Transformation to Earth Axes

$$\begin{aligned} \dot{\gamma} &= q_w \cos\phi - \frac{g}{V_0} \sin^2\phi \\ \dot{\psi} &= q_w \sin\phi + \frac{g}{V_0} \sin\phi \cos\phi \end{aligned} \tag{17}$$

where q_w is wind-referenced pitch rate and p and q are body-referenced roll and pitch rates, ϕ is roll, δ_e is elevator deflection, δ_a is aileron deflection, g is the gravitational constant, V_0 is airspeed, and Z_α, L_p, etc., are constants (the constants $\bar{Z}_\alpha$, $\bar{L}_p$, etc., defined in the appendix) related to the stability derivatives of the airframe. Notice that the first equation in Eq. (16) depends on $\dot{\delta}_e$. Therefore, $\dot{\delta}_e$ and δ_a will be treated as the control variables, while δ_e will be considered to be a state variable in the derivation of a decoupling control law. Such a control law can be implemented by simply placing an integrator in the elevator control loop as shown in Fig. 1.

For convenience, we will use a two-step design procedure. We will first design an inner-loop which decouples q_w from ϕ, and then using the decoupled inner-loop we will design an outer-loop which decouples γ from ψ.

Inner-Loop Design

The aircraft equations of motion in Eq. (16) which are the equations of the open inner-loop are rewritten here using matrix notation.

$$\begin{bmatrix} \dot{q}_w \\ \dot{q} \\ \dot{p} \\ \dot{\phi} \\ \dot{\delta}_e \end{bmatrix} = \begin{bmatrix} Z_\alpha(q - q_w) + \frac{g}{V_0} p \sin\phi \\ M_q q + M_{qw} q_w + M_\phi(\cos\phi - 1) + M_{\delta_e}\delta_e \\ L_p p + L_\phi \sin\phi \\ p \\ 0 \end{bmatrix} + \begin{bmatrix} -Z_{\delta_e} & 0 \\ 0 & 0 \\ 0 & L_{\delta_a} \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\delta}_e \\ \delta_a \end{bmatrix} \tag{18}$$

where the output vector; $y^T = [q_w, \phi]$ is related to the state vector add

$$y = \begin{bmatrix} q_w \\ \phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} x \tag{19}$$

First, we construct the matrix $A_1(x)$ according to Eq. (3)

$$A_1(x) = \begin{bmatrix} -Z_\alpha & Z_\alpha & \frac{g}{V_0}\sin\phi & \frac{g}{V_0}p\cos\phi & 0 \\ M_{qw} & M_q & 0 & -M_\phi \sin\phi & M_{\delta_e} \\ 0 & 0 & L_p & L_\phi \cos\phi & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and compute the parameters $d_1 = 0$, $d_2 = 1$. Using these parameters we then form the matrices $D(x)$ and $f^*(x)$ according to Eq. (4).

$$D(x) = \begin{bmatrix} C_1 B(x) \\ C_2 A_1(x) B(x) \end{bmatrix} = \begin{bmatrix} -Z_{\delta_e} & 0 \\ 0 & L_{\delta_a} \end{bmatrix};$$

$$f^*(x) = \begin{bmatrix} Z_\alpha(q - q_w) + \frac{g}{V_0}p\sin\phi \\ L_p p + L_\phi \sin\phi \end{bmatrix} \tag{20}$$

$D(x)$ is nonsingular for all $x \in R^n$, therefore, the system can be decoupled everywhere in the state space. The control law for $\dot{\delta}_e$ and δ_a which decouples the inner-loop follows directly from Eqs. (2, 7, and 20).

$$\begin{aligned} \dot{\delta}_e &= \frac{1}{Z_{\delta_e}}[Z_\alpha(q - q_w) + \frac{g}{V_0}p\sin\phi - w_1] \\ \delta_a &= -\frac{1}{L_{\delta_a}}[L_p p + L_\phi \sin\phi - w_2] \end{aligned} \tag{21}$$

In this equation w_1 and w_2 are inner-loop command inputs which are internally generated by the flight path control system. The closed-loop dynamics of the inner-loop are obtained by substituting the control law, Eq. (21) into Eq. (16). The resulting equation

$$\begin{bmatrix} \dot{q}_w \\ \dot{q} \\ \dot{p} \\ \dot{\phi} \\ \dot{\delta}_e \end{bmatrix} = \begin{bmatrix} w_1 \\ M_q q + M_{qw} q_w + M_\phi(\cos\phi - 1) + M_{\delta_e}\delta_e \\ w_2 \\ p \\ \frac{1}{Z_{\delta_e}}[Z_\alpha(q - q_w) + \frac{g}{V_0}p\sin\phi - w_1] \end{bmatrix} \tag{22}$$

clearly shows that the output variables of the inner-loop q_w and ϕ are decoupled while the state variables q and δ_e remain coupled to each other and to both q_w and ϕ.

Outer-Loop Design

The objective of the outer-loop design is to find the control law for w_1 and w_2 that will decouple the vertical (γ) and the horizontal (ψ) flight path angles. The state vector for the outer-loop is $x^T = [\phi, p, q_w, \gamma, \psi]$. Thus, using Eqs. (17) and (22), the outer-loop dynamics can be represented by the following equation

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \\ \dot{q}_w \\ \dot{\gamma} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p \\ 0 \\ 0 \\ -\frac{g}{V_0}\sin^2\phi + q_w\cos\phi \\ \frac{g}{2V_0}\sin 2\phi + q_w \sin\phi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\begin{bmatrix} \gamma \\ \psi \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x \tag{23}$$

Now using Eqs. (3) and (4) one gets a matrix $D(x)$

$$D(x) = CA_1(x)B = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \end{bmatrix}$$

which is singular for all $x \in R^n$. Therefore, the outer-loop representation in Eq. (23) is not compatible with decoupling. In order to obtain a more convenient representation for the outer-loop we let w_3 ($= \dot{w}_1$) and w_2 be the control inputs and we treat w_1 as a state variable. The resulting outer-loop state vector is $x^T = [\phi, p, q_w, w_1, \gamma, \psi,]$ and the outer-loop dynamics are given by

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \\ \dot{q}_w \\ \dot{w}_1 \\ \dot{\gamma} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p \\ 0 \\ w_1 \\ 0 \\ -\frac{g}{V_0}\sin^2\phi + q_w\cos\phi \\ \frac{g}{2V_0}\sin 2\phi + q_w\sin\phi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_2 \\ w_3 \end{bmatrix}$$

$$\begin{bmatrix} \gamma \\ \psi \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x \tag{24}$$

The matrices $CA_1(x)$ and $CA_2(x)$ which are of interest here are constructed from Eq. (3) as follows

$$CA_1(x) = \begin{bmatrix} -\xi & 0 & \cos\phi & 0 & 0 & 0 \\ \zeta & 0 & \sin\phi & 0 & 0 & 0 \end{bmatrix}$$

$$CA_2(x) = \begin{bmatrix} -\mu & -\xi & -p\sin\phi & \cos\phi & 0 & 0 \\ \nu & \zeta & p\cos\phi & \sin\phi & 0 & 0 \end{bmatrix} \tag{25}$$

where

$$\begin{aligned} \xi &= q_w \sin\phi + \frac{g}{V_0} \cdot \sin 2\phi \\ \zeta &= q_w \cos\phi + \frac{g}{V_0} \cdot \cos 2\phi \end{aligned} \tag{26a}$$

$$\begin{aligned} \mu &= \dot{\xi} = (q_w \cos\phi + 2\frac{g}{V_0}\cos 2\phi)p + w_1 \sin\phi \\ \nu &= \dot{\zeta} = (q_w \sin\phi + 2\frac{g}{V_0}\sin 2\phi)p + w_1\cos\phi \end{aligned} \tag{26b}$$

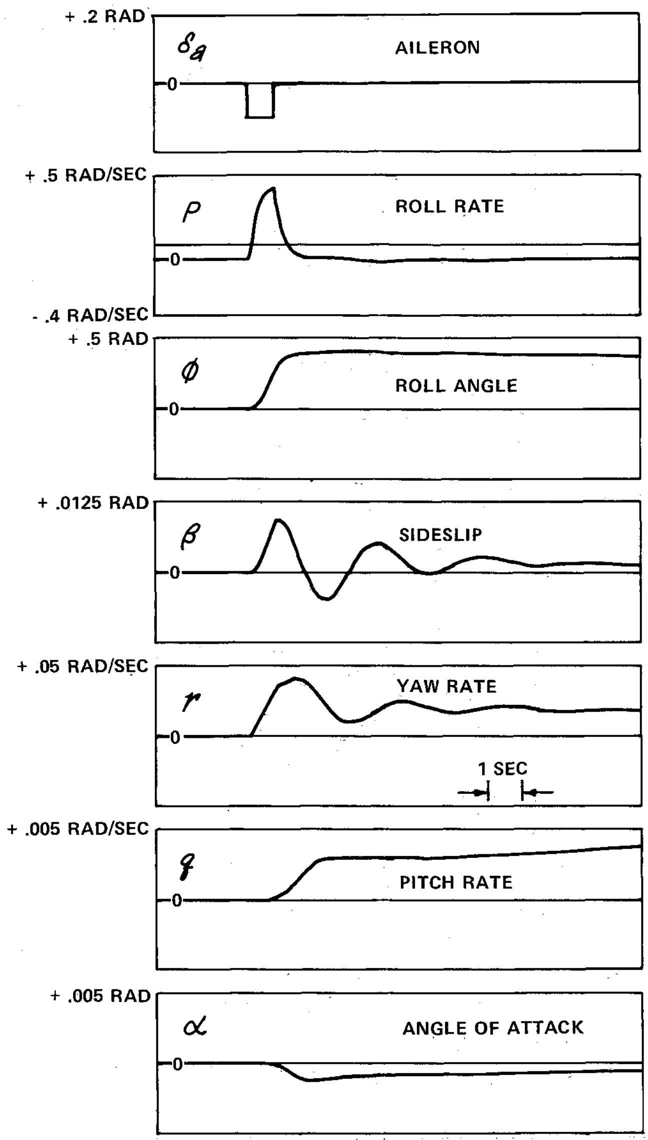

Fig. 2 Real aircraft response to pulse aileron.

We now compute the parameters d_i ($d_1 = d_2 = 2$) and the matrices $D(x)$ and $f^*(x)$ defined in Eq. (4)

$$D(x) = \begin{bmatrix} -\xi & \cos\phi \\ \zeta & \sin\phi \end{bmatrix} ; f^*(x) = CA_2(x)f(x)$$

$$= \begin{bmatrix} -\mu p - pw_1 \sin\phi \\ \nu p + pw_1 \cos\phi \end{bmatrix} \tag{27}$$

The determinant of D is $|D| = -(q_w + (g/V_0)\cos\phi)$, hence, $|D|$ is nonsingular everywhere in the state space except on the cylindrical surface $q_w + (g/V_0)\cos\phi = 0$. This surface has a specific physical significance. Since $|D|$ is proportional to the total normal (to the wings) acceleration $V_0 q_w + g\cos\phi$ it represents the set of operating points which correspond to zero gravity condition. Therefore, the flight condition for which decoupling breaks down is a flight condition that must be avoided for several reasons. A zero gravity condition can be prevented by simply limiting w_2 as a function of w_3. This will in turn tend to limit q_w and ϕ in such a way as to maintain $|D| \neq 0$ at all times. The complete structure of the control law is given by Eq. (11) where K and Λ and v are chosen as

$$K = \begin{bmatrix} -a_1 & -b_1 & -c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_2 & -b_2 & -c_2 \end{bmatrix} ;$$

$$\Lambda = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} ; v = \begin{bmatrix} \gamma_c \\ \psi_c \end{bmatrix} \tag{28}$$

where γ_c and ψ_c are the vertical and horizontal flight path angle command inputs. Substituting Eqs. (27) and (28) into Eq. (14) yields the control law for w_2 and w_3.

$$\begin{bmatrix} w_2 \\ w_3 \end{bmatrix} = \frac{1}{|D|} \begin{bmatrix} \sin\phi & -\cos\phi \\ -\zeta & -\xi \end{bmatrix} \begin{bmatrix} -(a_1\gamma + b_1\dot\gamma + c_1\ddot\gamma - a_1\gamma_c) + \mu p + pw_1\sin\phi \\ -(a_2\psi + b_2\dot\psi + c_2\ddot\psi - a_2\psi_c) - \nu p - pw_1\cos\phi \end{bmatrix} \tag{29}$$

$\dot\gamma$, $\ddot\gamma$, $\dot\psi$ and $\ddot\psi$ have been retained and the result is left in matrix form in order to conserve space. Ordinarily, the control law is expressed as a function of the state variables. This is done by substituting the right hand side of Eq. (17) for $\dot\gamma$ and $\dot\psi$ and substituting the following expression for $\ddot\gamma$ and $\ddot\psi$.

$$\begin{aligned} \ddot\gamma &= -\xi p + w_1\cos\phi \\ \ddot\psi &= \zeta p + w_1\sin\phi \end{aligned} \tag{30}$$

Equation (29) is in a particularly useful form for deriving an expression for the closed-loop dynamics. We differentiate Eq. (30) and use Eqs. (26) and (29) and substitute w_3, w_1, w_2 for $\dot w_1$, $\dot q_w$, $\dot p$, respectively. The resulting expression

$$\begin{aligned} \dddot\gamma + c_1\ddot\gamma + b_1\dot\gamma + a_1(\gamma - \gamma_c) &= 0 \\ \dddot\psi + c_2\ddot\psi + b_2\dot\psi + a_2(\psi - \psi_c) &= 0 \end{aligned} \tag{31}$$

clearly shows that the ultimate output variables γ and ψ have been decoupled. These variables evolve according to linear differential equations while the state variables ϕ, p, q_w and w_1 obey a nonlinear differential relationship [just substitute Eq. (29) into Eq. (24)] and respond to both command inputs. The block diagram of the control system is shown in Fig. 1. The inner-loop control law [Eq. (27)] generates the control inputs δ_e and δ_a. δ_a is fed directly to the aileron while an integrator is placed between the control law for δ_e and the elevator. A similar procedure is used to generate the inner-loop command inputs w_1, w_2 from the outer-loop controls w_3, w_2.

Outer-loop decoupling could have been achieved by replacing the relationship $\dot p = w_2$ in Eq. (23) by $\phi = w_4$, where w_4 is a new command input defined by $\dot w_4 = w_2$. This method reduces the dimension of the outer-loop state vector to four, $x^T = [\phi, q_w, \gamma, \psi]$. Since the outer-loop control law in this case will generate w_1 and w_4, w_4 must be differentiated to yield the inner-loop command input w_2. But w_4 depends on the command inputs γ_c and ψ_c, which need not be differentiable. Therefore, the design procedure just described is not completely valid from a theoretical viewpoint. In practice, however, it may be possible to implement this control law, which turns out to be simpler than the one presented in Eq. (25), by using an approximate differentiator.

Simulation Results

The real aircraft Eqs. (A1–A3) and the inner and outer loop control laws, Eqs. (21) and (29) were

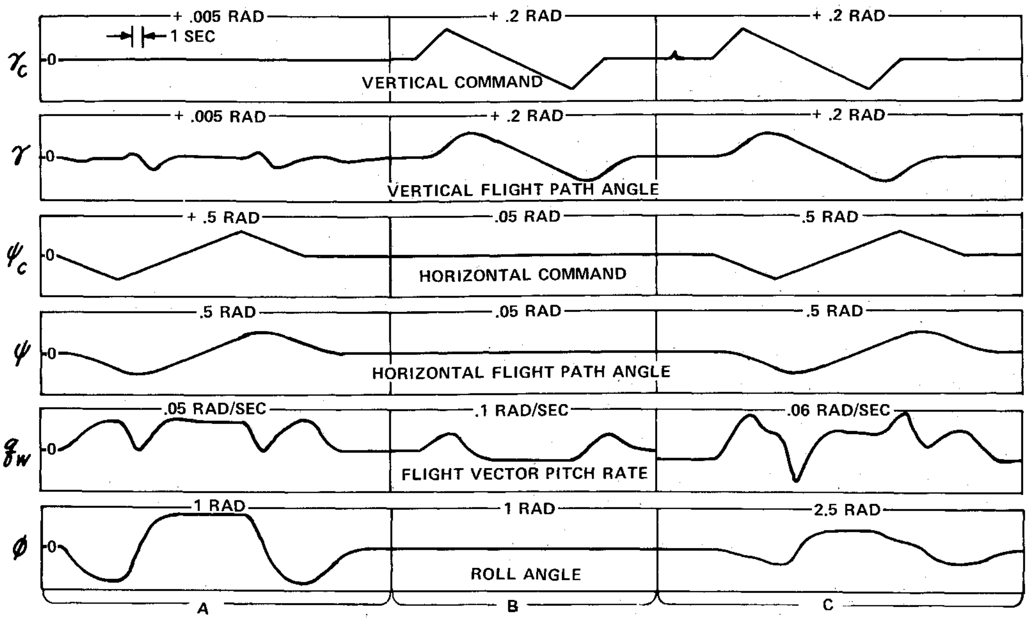

Fig. 3 Flight vector response to ramp inputs.

programmed on a computer in order to evaluate the decoupling performance in a real environment. The results of the simulation study can be summarized as follows:

1) The rudder control law effectively maintained a very small sideslip angle in the entire flight regime and increased the dutch-roll damping of the open-loop airframe.

2) The responses of the vertical and horizontal flight path angles were stable and decoupled.

3) The control system design was not sensitive to variations in aircraft parameters.

4) Airspeed variations did not appreciably degrade decoupling performance. [Airspeed variations were induced by adding the drag equation to the 5 degrees of freedom aircraft equations in Eq. (A1).

Figure 2 shows the open-loop aircraft response to a pulse aileron input at wings level. A pulse aileron input produces a constant bank angle and a loss in vertical lift which shows up in the figure as a reduction in the angle of attack. The combined effect of a bank angle and the loss of lift produces a change in both the vertical and horizontal flight path angles. The variation in q is due to the nonzero component of the angular velocity vector along the y-body axis. Notice from the β and r time histories that the dutch-roll mode of the open-loop aircraft is underdamped.

Figure 3 shows the response of the closed-loop aircraft to pilot command inputs. Part a) of the figure shows the aircraft response to a horizontal command input consisting of a right turn followed by a left turn and a right turn. Part b) of the figure shows the aircraft response to a vertical command input consisting of a pull-up followed by a push-over and a pull-up. Notice that only one flight path angle responds to a given command input in each figure. When the command inputs are combined in part c) of the figure each channel responds as if no input was applied to the other channel and thus the figure conclusively demonstrates decoupling. An underdamped oscillation was observed in the body referenced pitch rate (q) and elevator (δ_e) responses during the simulation study. Damping was introduced by adding q feedback to δ_e which produced a small coupling in a perfectly decoupled inner-loop. The response shown in Fig. 3 was obtained by introducing 10% change in the real aircraft parameters to determine the sensitivity of the control system design. The coupling observed in part a) of the figure was caused by the combined effect of using the ideal aircraft control law to decouple the real aircraft, introducing inner-loop coupling and offsetting the aircraft parameters. The coupling for 40° roll angle is only 0.05° of γ, or γ/ϕ = 0.00125, which shows indeed that the control system produces excellent decoupling.

Conclusions

A necessary and sufficient condition for decoupling the class of nonlinear systems defined in Eq. (1) has been obtained. It is shown that if the matrix $D(x)$ is nonsingular in the domain $X \in R^n$ then there exists a control law that decouples the system for all $x \in X$. The results obtained here are similar to those in Singh and Rugh[10] where a more thorough analysis of the nonlinear decoupling problem is given. Although the invertibility of $D(x)$ is essential to decoupling, in many instances a singular $D(x)$ simply implies that the specific mathematical model used to represent the physical process is not compatible with decoupling. In such instances, using a more convenient representation of the system (i.e., by letting δ_e be the state and $\dot{\delta}_e$ be the control) usually leads to decoupling. This was the case in the aircraft control problem discussed above.

The extended decoupling method[5,6] provides decoupling and at the same time insures closed-loop stability in a linear system. A similar approach does not exist for nonlinear systems. Therefore, the process of decoupling a nonlinear system may lead to unstable decoupled system or may result in a closed-loop system with undesirable dynamic characteristics. When the criterion $\Sigma_{i=1}^{m}(1 + d_i) = n$ is satisfied there is a strong possibility that the decoupling control law will yield a stable closed-loop system, otherwise there is no guarantee that the decoupled closed-loop will be stable. In the aircraft control problem the outer-loop satisfies this criterion while the inner-loop does not. As a result, an underdamped pitch rate response was observed during the simulation runs. However, this response remained localized in the inner-loop and did not affect the decoupled response of the outer-loop.

Appendix: Aircraft Dynamics in Turning Flight

The effect of the large roll motion in a banked turn can be adequately modeled by including nonlinear roll terms in a basically linear aircraft model.[11] The aircraft model used here is based on the following simplifying assumptions: 1) Inertial coupling is negligible, 2) Airspeed is constant, 3) Aerodynamic forces can be linearized about wings-level flight at constant altitude and constant airspeed to yield a linear set of equations of motion, and 4) The bank angle cross-coupling that results from non-wings level flight can be modeled by including nonlinear terms associated with roll angle.

The aircraft equations of motion relative to body axes are given by

Sideslip

$$\dot\beta = -r + \alpha_0 p + \frac{g}{V_0}\sin\phi + [Y_{\delta_r}\delta_r - Y_{\dot\beta}\dot\beta - Y_p p - Y_r r - Y_\beta \beta] \quad \text{(A1a)}$$

Lift Force

$$\dot\alpha = q - Z_\alpha \alpha - \frac{g}{V_0}(1 - \cos\phi) + Z_{\delta_e}\delta_e \quad \text{(A1b)}$$

Rolling Moment

$$\dot p = L_p p + L_r r + L_{\delta_a}\delta_a + [L_{\dot\beta}\dot\beta + L_\beta\beta + L_{\dot r}\dot r + L_{\delta_r}\delta_r] \quad \text{(A1c)}$$

Pitching Moment

$$\dot q = -M_q q - M_\alpha \alpha - M_{\dot\alpha}\dot\alpha + M_{\delta_e}\delta_e \quad \text{(A1d)}$$

Yawing Moment

$$\dot r = N_r r + N_p p + N_\beta \beta + N_{\dot p}\dot p + N_{\dot\beta}\dot\beta + N_{\delta_r}\delta_r + N_{\delta_a}\delta_a \quad \text{(A1e)}$$

where p, q, r are roll, pitch, and yaw rates in body axes, α is the angle of attack, β is sideslip, ϕ is roll angle, δ_a, δ_e and δ_r are aileron, elevator and rudder deflections. α_0 and V_0 are trim angle of attack and airspeed. The capital letters are the dimensional stability derivatives normalized with respect to mass and inertia. The nonlinear roll term in Eq. (A1b) represents the loss of vertical life during a banked turn.

Suppose that the aircraft is climbing while in a banked turn. The resultant angular velocity vector can be resolved along two sets of orthogonal axes. The components along the horizontal and vertical axes are $\dot\gamma$ and $\dot\psi$ and the components perpendicular and parallel to the wings are q_w and r_w. These components are related by the Euler transformation

$$\begin{aligned} \dot\psi &= q_w \sin\phi + r_w \cos\phi \\ \dot\gamma &= q_w \cos\phi - r_w \sin\phi \\ \dot\phi &= p \end{aligned} \quad \text{(A2)}$$

q_w and r_w on the other hand are related to the body rates q and r by

$$q_w = q - \dot\alpha \quad \text{(A3a)}$$

$$r_w = r - \alpha_0 p + \dot\beta \quad \text{(A3b)}$$

Equations (A1–A3) adequately represent the motion of a "real" aircraft in turning flight relative to earth axes. A reasonable simplification can be made in these equations by postulating a rudder control law which maintains a negligibly small sideslip angle during the entire flight regime.

$$\delta_r = \frac{1}{Y_{\delta_r}}[Y_p p + Y_r r + Y_\beta \beta] \quad \text{(A4)}$$

This control law also introduces damping into the dutch-roll mode. Moreover, it provides the following algebraic relation between roll rate p and yaw rate r

$$r_w = r - \alpha_0 p + \dot\beta = \frac{g}{V_0}\sin\phi \quad \text{(A5)}$$

Because of this relation, r is no longer an independent degree of freedom and hence the lateral dynamics of the aircraft depend only on the rolling moment (differential) equation. The latter is obtained as follows: substitute the rudder control law, Eq. (A4) into Eqs. (A1c) and (A1e), eliminate the terms $\dot p$ and $\dot r$ from the right-hand side of these equations, substitute Eq. (A5) into the resulting equations, set $\beta = \dot\beta = 0$, and omit the yawing moment equation.

The result is

$$\dot p = \bar L_p p + \bar L_\phi \sin\phi + \bar L_{\delta_a}\delta_a \quad \text{(A6)}$$

$$\bar L_\phi = \frac{g}{V_0}\left[L_r + Y_r \frac{L_{\delta r}}{Y_{\delta r}} + L_{\dot r}\left(N_r + Y_r \frac{N_{\delta r}}{Y_{\delta r}}\right)\right]$$

where

$$\bar L_{\delta_a} = L_{\delta_a} + L_{\dot r} N_{\delta a}$$

$$\bar L_p = (L_p + \alpha_0 L_r) + L_{\dot r}(N_p + \alpha_0 N_r) + (Y_p + \alpha_0 Y_r)(L_{\delta_r} + L_{\dot r} N_{\delta_r})/Y_{\delta_r}$$

The lift force and pitching moment equations can be manipulated into a convenient form for decoupling. Since the Euler equations (A2) are expressed in terms of q_w, it is convenient to eliminate α from these equations. This is done by solving for α from Eq. (A1b) and substituting the result into Eq. (A1d) and then eliminating $\dot\alpha$ from Eq. (A1d) and Eq. (A3a). The result is

$$\dot q = \bar M_q q + \bar M_{qw} q_w + \bar M_\phi (\cos\phi - 1) + \bar M_{\delta e}\delta_e$$

where

$$\begin{aligned} \bar M_q &= -M_q - M_{\dot\alpha} \\ \bar M_{qw} &= M_{\dot\alpha} - M_\alpha / Z_\alpha \\ \bar M_\phi &= -g M_\alpha / V_0 Z_\alpha \\ \bar M_{\delta e} &= M_{\delta e} - M_\alpha / Z_{\delta_e} / Z_\alpha \end{aligned} \quad \text{(A7)}$$

Equation (A1b) is now differentiated and $\dot\alpha$ is eliminated through Eq. (A3a). The resulting equation is

$$\dot q_w = Z_\alpha (q - q_w) - Z_{\delta_e}\dot\delta_e + \frac{g}{V_0}\dot\phi \sin\phi \quad \text{(A8)}$$

Finally, substituting Eq. (A5) into Eq. (A2) yields the following Euler transformation equations

$$\begin{aligned} \dot\gamma &= q_w \cos\phi - \frac{g}{V_0}\sin^2\phi \\ \dot\psi &= q_w \sin\phi + \frac{g}{V_0}\sin\phi \cos\phi \end{aligned} \quad \text{(A9)}$$

Equations (A6–A9) now describe the motion of an "ideal" aircraft which maintains zero sideslip during the entire flight regime.

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Computer Aided Design-Drafting (CADD)—Engineering/Manufacturing Tool

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A description is given of a powerful computer-operated graphic system, used from design through manufacture at McDonnell Aircraft Company (MCAIR). This system has made designers many times more productive than when they are using conventional drawing board methods. High engineering productivity, however, is only an initial benefit. When fully developed, the system will allow Manufacturing to machine parts, utilizing the programmed data created by the designer at the console, without writing additional programmed instructions to drive the milling machines. In addition, Tool Design and Quality Assurance have direct access to the original three-dimensional geometric data, thus eliminating misinterpretation of design intentions. With lofted surfaces developed, defined mathematically, and stored in a shared-computer file, a designer is able to indicate the plane in which a lofted contour is desired, and in a matter of seconds, he can have the contour displayed on the CRT. This enables the designer to create a design almost as fast as he can think. Further, his designs are defined mathematically at a degree of accuracy never before known. Other disciplines which interface with design are strength, aerodynamics, thermodynamics, and propulsion. With continued use and development of this system, even greater time and cost-saving techniques will be realized.

Introduction

ANY business firm, and especially a company oriented to the aerospace industry, must continually seek methods for improving its varied functions if it is to retain its competitive position. Such improvements are normally measured in terms of reduced costs, reduced manhours, reduced lead time, and better products.

The design of an aerospace vehicle, by necessity, is dictated by the many parametric requirements of each of the systems and technical disciplines comprising the total design effort. In order that each discipline may be properly interfaced with the other disciplines, and the integrity of the vehicle performance and specification requirements maintained, very close intergroup coordination is required. During any program, several design iterations to system components are usually required as the result of loads analyses, weight and balance requirements, aerodynamic considerations, and other design ramifications. As a result of these necessary design iterations, there is often some degree of difficulty encountered in effecting a smooth and orderly response to the required realignment of the specific designs to meet the new requirements. Reaction time by all affected parties must be coordinated closely so that the major milestones are not jeopardized. Although many design changes are minor in nature, some can be very complex. Changes of this type can be extremely disruptive and require that every available technique, talent, and design tool at our disposal be directed toward a common goal of producing the necessary changes on a timely basis to assure a good design on schedule. Computer Aided Design-Drafting, or CADD (pronounced caddy) as it is commonly called at MCAIR, is a tool that we are presently using to help us meet that goal.

The term, CADD, denotes various computer techniques and applications where data are either presented or accepted by a computer in the form of line drawings or graphs, as opposed to alphanumerics only. CADD is "interactive," which implies that there is an efficient and real-time interplay of actions between the console operator and the system hardware devices. CADD therefore describes an interactive and conversational mode of operation, utilizing a display console where the engineer may describe his design, perform analysis procedures, and make changes to the design if he so chooses.

A designer is normally concerned with creating a geo-

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